単回帰曲線の求め方

ここでは例として、2次曲線の一般型として次の曲線から考えます。

$y = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + b_5x^5 \ (= f(x))$

この曲線について最小2乗法を当てはめていきます。

実際の値と曲線との距離 $L = |\ y_i - f(x_i)\ |$ の2乗和の最小値を求める。

実際には、偏微分

$\partial L / \partial b_0 = 0,\ \partial L / \partial b_1 = 0,\ \partial L / \partial b_2 = 0,\ \partial L / \partial b_3 = 0,\ \partial L / \partial b_4 = 0,\ \partial L / \partial b_5 = 0$

を解けばよい。

$$\frac{\partial L}{\partial b_0} = \frac{\partial}{\partial b_0} \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$
$$\frac{\partial L}{\partial b_1} = \frac{\partial}{\partial b_1} \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$
$$\frac{\partial L}{\partial b_2} = \frac{\partial}{\partial b_2} \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$
$$\frac{\partial L}{\partial b_3} = \frac{\partial}{\partial b_3} \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$
$$\frac{\partial L}{\partial b_4} = \frac{\partial}{\partial b_4} \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$
$$\frac{\partial L}{\partial b_5} = \frac{\partial}{\partial b_5} \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$

$$-2 \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5) = 0$$
$$-2 \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)\ x_i = 0$$
$$-2 \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)\ x_i^2 = 0$$
$$-2 \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)\ x_i^3 = 0$$
$$-2 \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)\ x_i^4 = 0$$
$$-2 \Sigma (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)\ x_i^5 = 0$$

$$\Sigma y_i = nb_0 + b_1 \Sigma x_i + b_2 \Sigma x_i^2 + b_3 \Sigma x_i^3 + b_4 \Sigma x_i^4 + b_5 \Sigma x_i^5 \quad \cdots ①$$
$$\Sigma x_iy_i = b_0 \Sigma x_i + b_1 \Sigma x_i^2 + b_2 \Sigma x_i^3 + b_3 \Sigma x_i^4 + b_4 \Sigma x_i^5 + b_5 \Sigma x_i^6 \quad \cdots ②$$
$$\Sigma x_i^2y_i = b_0 \Sigma x_i^2 + b_1 \Sigma x_i^3 + b_2 \Sigma x_i^4 + b_3 \Sigma x_i^5 + b_4 \Sigma x_i^6 + b_5 \Sigma x_i^7 \quad \cdots ③$$
$$\Sigma x_i^3y_i = b_0 \Sigma x_i^3 + b_1 \Sigma x_i^4 + b_2 \Sigma x_i^5 + b_3 \Sigma x_i^6 + b_4 \Sigma x_i^7 + b_5 \Sigma x_i^8 \quad \cdots ④$$
$$\Sigma x_i^4y_i = b_0 \Sigma x_i^4 + b_1 \Sigma x_i^5 + b_2 \Sigma x_i^6 + b_3 \Sigma x_i^7 + b_4 \Sigma x_i^8 + b_5 \Sigma x_i^9 \quad \cdots ⑤$$
$$\Sigma x_i^5y_i = b_0 \Sigma x_i^5 + b_1 \Sigma x_i^6 + b_2 \Sigma x_i^7 + b_3 \Sigma x_i^8 + b_4 \Sigma x_i^9 + b_5 \Sigma x_i^{10} \quad \cdots ⑥$$

これらの ①②③④⑤⑥ の連立方程式を解けば、$b_0, b_1, b_2, b_3, b_4, b_5$ の値がわかるので、5次の回帰曲線が特定できる。