

単回帰曲線の求め方

ここでは例として、2次曲線の一般型として次の曲線から考えます。

$$y = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + b_5x^5 (= f(x))$$

この曲線について最小2乗法を当てはめていきます。

実際の値と曲線との距離 $L = |y_i - f(x_i)|$ の2乗和の最小値を求める。

実際には、偏微分

$L / b_0 = 0, L / b_1 = 0, L / b_2 = 0, L / b_3 = 0, L / b_4 = 0, L / b_5 = 0$
を解けばよい。

$$\frac{\partial L}{\partial b_0} = \frac{\partial}{\partial b_0} (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$

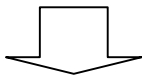
$$\frac{\partial L}{\partial b_1} = \frac{\partial}{\partial b_1} (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$

$$\frac{\partial L}{\partial b_2} = \frac{\partial}{\partial b_2} (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$

$$\frac{\partial L}{\partial b_3} = \frac{\partial}{\partial b_3} (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$

$$\frac{\partial L}{\partial b_4} = \frac{\partial}{\partial b_4} (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$

$$\frac{\partial L}{\partial b_5} = \frac{\partial}{\partial b_5} (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5)^2 = 0$$



$$-2 (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5) = 0$$

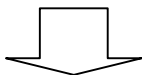
$$-2 (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5) x_i = 0$$

$$-2 (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5) x_i^2 = 0$$

$$-2 (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5) x_i^3 = 0$$

$$-2 (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5) x_i^4 = 0$$

$$-2 (y_i - b_0 - b_1x_i - b_2x_i^2 - b_3x_i^3 - b_4x_i^4 - b_5x_i^5) x_i^5 = 0$$



$$y_i = nb_0 + b_1 x_i + b_2 x_i^2 + b_3 x_i^3 + b_4 x_i^4 + b_5 x_i^5 \quad \dots$$

$$x_i y_i = b_0 x_i + b_1 x_i^2 + b_2 x_i^3 + b_3 x_i^4 + b_4 x_i^5 + b_5 x_i^6 \quad \dots$$

$$x_i^2 y_i = b_0 x_i^2 + b_1 x_i^3 + b_2 x_i^4 + b_3 x_i^5 + b_4 x_i^6 + b_5 x_i^7 \quad \dots$$

$$x_i^3 y_i = b_0 x_i^3 + b_1 x_i^4 + b_2 x_i^5 + b_3 x_i^6 + b_4 x_i^7 + b_5 x_i^8 \quad \dots$$

$$x_i^4 y_i = b_0 x_i^4 + b_1 x_i^5 + b_2 x_i^6 + b_3 x_i^7 + b_4 x_i^8 + b_5 x_i^9 \quad \dots$$

$$x_i^5 y_i = b_0 x_i^5 + b_1 x_i^6 + b_2 x_i^7 + b_3 x_i^8 + b_4 x_i^9 + b_5 x_i^{10} \quad \dots$$

これらの 連立方程式を解けば、 $b_0, b_1, b_2, b_3, b_4, b_5$ の値がわかるので、5次の回帰曲線が特定できる。