

重回帰式の求め方

ここでは例として、目的変数が1つ、説明変数が6つの以下のものを考えます。

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_6x_6 \quad (= f(x_1, x_2, x_3, x_4, x_5, x_6))$$

この回帰式について最小2乗法を当てはめていきます。

実際の値と回帰式との距離 $L = |y_i - f(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i})|$ の2乗和の最小値を求める。

実際には、偏微分

$$L / b_0 = 0, \quad L / b_1 = 0, \quad L / b_2 = 0, \quad L / b_3 = 0,$$

$$L / b_4 = 0, \quad L / b_5 = 0, \quad L / b_6 = 0$$

を解けばよい。

$$\frac{L}{b_0} = \frac{1}{b_0} (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$

$$\frac{L}{b_1} = \frac{1}{b_1} (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$

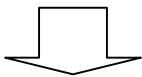
$$\frac{L}{b_2} = \frac{1}{b_2} (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$

$$\frac{L}{b_3} = \frac{1}{b_3} (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$

$$\frac{L}{b_4} = \frac{1}{b_4} (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$

$$\frac{L}{b_5} = \frac{1}{b_5} (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$

$$\frac{L}{b_6} = \frac{1}{b_6} (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$



$$-2 (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) = 0$$

$$-2 (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{1i} = 0$$

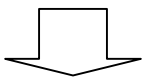
$$-2 (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{2i} = 0$$

$$-2 (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{3i} = 0$$

$$-2 (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{4i} = 0$$

$$-2 (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{5i} = 0$$

$$-2 (y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) x_{6i} = 0$$



$$y_i = nb_0 + b_1 x_{1i} + b_2 x_{2i} + b_3 x_{3i} + b_4 x_{4i} + b_5 x_{5i} + b_6 x_{6i} \quad \dots$$

$$x_{1i}y_i = b_0 x_{1i} + b_1 x_{1i}x_{1i} + b_2 x_{1i}x_{2i} + b_3 x_{1i}x_{3i} + b_4 x_{1i}x_{4i} + b_5 x_{1i}x_{5i} + b_6 x_{1i}x_{6i} \quad \dots$$

$$x_{2i}y_i = b_0 x_{2i} + b_1 x_{2i}x_{1i} + b_2 x_{2i}x_{2i} + b_3 x_{2i}x_{3i} + b_4 x_{2i}x_{4i} + b_5 x_{2i}x_{5i} + b_6 x_{2i}x_{6i} \quad \dots$$

$$x_{3i}y_i = b_0 x_{3i} + b_1 x_{3i}x_{1i} + b_2 x_{3i}x_{2i} + b_3 x_{3i}x_{3i} + b_4 x_{3i}x_{4i} + b_5 x_{3i}x_{5i} + b_6 x_{3i}x_{6i} \quad \dots$$

$$x_{4i}y_i = b_0 x_{4i} + b_1 x_{4i}x_{1i} + b_2 x_{4i}x_{2i} + b_3 x_{4i}x_{3i} + b_4 x_{4i}x_{4i} + b_5 x_{4i}x_{5i} + b_6 x_{4i}x_{6i} \quad \dots$$

$$x_{5i}y_i = b_0 x_{5i} + b_1 x_{5i}x_{1i} + b_2 x_{5i}x_{2i} + b_3 x_{5i}x_{3i} + b_4 x_{5i}x_{4i} + b_5 x_{5i}x_{5i} + b_6 x_{5i}x_{6i} \quad \dots$$

$$x_{6i}y_i = b_0 x_{6i} + b_1 x_{6i}x_{1i} + b_2 x_{6i}x_{2i} + b_3 x_{6i}x_{3i} + b_4 x_{6i}x_{4i} + b_5 x_{6i}x_{5i} + b_6 x_{6i}x_{6i} \quad \dots$$

これらの 連立方程式 を解けば、 $b_0, b_1, b_2, b_3, b_4, b_5, b_6$ の値がわかるので、重回帰式が特定できる。