重回帰式の求め方

ここでは例として、目的変量が１つ、説明変量が６つの以下のものを考えます。

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_6x_6 (= f(x_1, x_2, x_3, x_4, x_5, x_6))$$

この回帰式について最小２乗法を当てはめていきます。
実際の値と回帰式との距離 $L = |y_i - f(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i})|$ の２乗和の最小値を求める。実際には、偏微分

$$\frac{\partial L}{\partial b_0} = 0,\ \frac{\partial L}{\partial b_1} = 0,\ \frac{\partial L}{\partial b_2} = 0,\ \frac{\partial L}{\partial b_3} = 0,\ \frac{\partial L}{\partial b_4} = 0,\ \frac{\partial L}{\partial b_5} = 0,\ \frac{\partial L}{\partial b_6} = 0$$

を解けばよい。

$$\frac{\partial L}{\partial b_0} = \frac{\partial}{\partial b_0}\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_1} = \frac{\partial}{\partial b_1}\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_2} = \frac{\partial}{\partial b_2}\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_3} = \frac{\partial}{\partial b_3}\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_4} = \frac{\partial}{\partial b_4}\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_5} = \frac{\partial}{\partial b_5}\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$
$$\frac{\partial L}{\partial b_6} = \frac{\partial}{\partial b_6}\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})^2 = 0$$

偏微分すると、

$$2\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i}) = 0$$
$$2\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})\ x_{1i} = 0$$
$$2\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})\ x_{2i} = 0$$
$$2\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})\ x_{3i} = 0$$
$$2\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})\ x_{4i} = 0$$
$$2\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})\ x_{5i} = 0$$
$$2\sum_{i=0}^{\infty}(y_i - b_0 - b_1x_{1i} - b_2x_{2i} - b_3x_{3i} - b_4x_{4i} - b_5x_{5i} - b_6x_{6i})\ x_{6i} = 0$$

更に整理して、

$$\sum_{i=0}^{\infty}y_i = nb_0 + b_1\sum_{i=0}^{\infty}x_{1i} + b_2\sum_{i=0}^{\infty}x_{2i} + b_3\sum_{i=0}^{\infty}x_{3i} + b_4\sum_{i=0}^{\infty}x_{4i} + b_5\sum_{i=0}^{\infty}x_{5i} + b_6\sum_{i=0}^{\infty}x_{6i} \tag{1}$$
$$\sum_{i=0}^{\infty}x_{1i}y_i = b_0\sum_{i=0}^{\infty}x_{1i} + b_1\sum_{i=0}^{\infty}x_{1i}x_{1i} + b_2\sum_{i=0}^{\infty}x_{1i}x_{2i} + b_3\sum_{i=0}^{\infty}x_{1i}x_{3i} + b_4\sum_{i=0}^{\infty}x_{1i}x_{4i} + b_5\sum_{i=0}^{\infty}x_{1i}x_{5i} + b_6\sum_{i=0}^{\infty}x_{1i}x_{6i} \tag{2}$$
$$\sum_{i=0}^{\infty}x_{2i}y_i = b_0\sum_{i=0}^{\infty}x_{2i} + b_1\sum_{i=0}^{\infty}x_{2i}x_{1i} + b_2\sum_{i=0}^{\infty}x_{2i}x_{2i} + b_3\sum_{i=0}^{\infty}x_{2i}x_{3i} + b_4\sum_{i=0}^{\infty}x_{2i}x_{4i} + b_5\sum_{i=0}^{\infty}x_{2i}x_{5i} + b_6\sum_{i=0}^{\infty}x_{2i}x_{6i} \tag{3}$$
$$\sum_{i=0}^{\infty}x_{3i}y_i = b_0\sum_{i=0}^{\infty}x_{3i} + b_1\sum_{i=0}^{\infty}x_{3i}x_{1i} + b_2\sum_{i=0}^{\infty}x_{3i}x_{2i} + b_3\sum_{i=0}^{\infty}x_{3i}x_{3i} + b_4\sum_{i=0}^{\infty}x_{3i}x_{4i} + b_5\sum_{i=0}^{\infty}x_{3i}x_{5i} + b_6\sum_{i=0}^{\infty}x_{3i}x_{6i} \tag{4}$$
$$\sum_{i=0}^{\infty}x_{4i}y_i = b_0\sum_{i=0}^{\infty}x_{4i} + b_1\sum_{i=0}^{\infty}x_{4i}x_{1i} + b_2\sum_{i=0}^{\infty}x_{4i}x_{2i} + b_3\sum_{i=0}^{\infty}x_{4i}x_{3i} + b_4\sum_{i=0}^{\infty}x_{4i}x_{4i} + b_5\sum_{i=0}^{\infty}x_{4i}x_{5i} + b_6\sum_{i=0}^{\infty}x_{4i}x_{6i} \tag{5}$$
$$\sum_{i=0}^{\infty}x_{5i}y_i = b_0\sum_{i=0}^{\infty}x_{5i} + b_1\sum_{i=0}^{\infty}x_{5i}x_{1i} + b_2\sum_{i=0}^{\infty}x_{5i}x_{2i} + b_3\sum_{i=0}^{\infty}x_{5i}x_{3i} + b_4\sum_{i=0}^{\infty}x_{5i}x_{4i} + b_5\sum_{i=0}^{\infty}x_{5i}x_{5i} + b_6\sum_{i=0}^{\infty}x_{5i}x_{6i} \tag{6}$$
$$\sum_{i=0}^{\infty}x_{6i}y_i = b_0\sum_{i=0}^{\infty}x_{6i} + b_1\sum_{i=0}^{\infty}x_{6i}x_{1i} + b_2\sum_{i=0}^{\infty}x_{6i}x_{2i} + b_3\sum_{i=0}^{\infty}x_{6i}x_{3i} + b_4\sum_{i=0}^{\infty}x_{6i}x_{4i} + b_5\sum_{i=0}^{\infty}x_{6i}x_{5i} + b_6\sum_{i=0}^{\infty}x_{6i}x_{6i} \tag{7}$$

これらの (1),(2),(3),(4),(5),(6),(7) の連立方程式を解けば、$b_0, b_1, b_2, b_3, b_4, b_5, b_6$ の値がわかるので、重回帰式が特定できる。